Fuzzy Linear Problem Solution Using Lower Bound and Upper Bound Technique

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Abstract: **Fuzzy Linear problem is an important application of fuzzy set theory in decision making problems and the problems are related to linear programming with fuzzy variables. In this paper, we proposed a method for Fuzzy linear programming problems using lower bound of the objective function and upper bound of the objective function. One numerical example illustrated with the help of the proposed method.**

Keywords: **Fuzzy linear problems, simplex method, optimal solution, lower bound of objective function, upper bound of objective function.**

1. INTRODUCTION

The basic linear programming problem is to find the optimum values of a linear function under given constraints. Linear Programming is one of the most frequently used operations research technique in real time problems. Many real world problems can be transformed into LP model. LP requires much well-defined and precise data which involves high costs. In order to cut the information costs and to avoid unrealistic assumptions, we used fuzzy linear program.

The concept of fuzzy decision making was proposed by **Bellman**, **Zadeh**, in 1970. **Zimmermann** proposed the first method of fuzzy linear programming with several objective functions. **Maleki et al**., added processes to clear up linear programming issues with fuzzy variables. **Cadenas, Verdegay** have evolved the concept of the usage of fuzzy numbers in linear programming. **Jayalakshmi, Pandian** added a new approach wherein fuzzy hassle is decomposed into 3 crisp linear programming problems. **Lodwick, Bachman** have mentioned huge scale fuzzy feasible optimization issues.

Definition : If X={x} is a collection of objects denoted by x, then a fuzzy set A in X is a set of order pairs

A={x, μ_A (x): x ε X}, where μ_A (x) is called the "membership grade" of x in A

Definition : Let A=(a,b), B=(d,e) be two triangular fuzzy numbers then

(i) A+B = (a+d, b+e)

(ii) kA = (ka,kb), where k is a real number

Definition : Let A=(a,b) then (i) A is said to be positive if a>0, b>0

(ii) A is said to be integer if a≥0, b≥0

2. PROBLEM FORMULATION

The fuzzy LPP is formulated as underneath:

$$\text{Max } Z=\sum_{j=1}^{n} C_j X_j$$

$$\text{Subject to } \sum_{j=1}^{n} A_{ij} X_j \leq B_i \ (i \varepsilon N_m)$$

$$X_j \geq 0 \ (j \ \varepsilon \ N_n).$$

where $A_{ij,}$ B_i, C_j are fuzzy numbers, and X_j are variables whose states are fuzzy numbers (iεN_m, jεN_n); the operations of addition and multiplication are operations of fuzzy arithmetic and ≤ denote the ordering of fuzzy members.

In this case, fuzzy numbers B_i ($i \varepsilon N_m$) typically have the form

$$B_i(x) = \begin{cases} 1 & when\ x \le b_i \\ \dfrac{b_i + p_i - x}{p_i} & when\ b_i < x < b_i + p_i \\ 0 & when\ b_i + p_i \le x \end{cases}$$

where $x \varepsilon R$.

Next, we determine the optimal values, by calculating both the lower bound and upper bounds.

Solving LPP to obtain optimal value of z1:

Max Z= cx

subject to $\sum_{j=1}^{n} a_{ij}x_j \le b_i$ ($i \varepsilon N_m$)

$x_j \ge 0$ ($j \varepsilon N_n$).

Solving LPP to obtain optimal values of z2: replace b_i with $b_i + p_i$,

Max Z=cx

subject to $\sum_{j=1}^{n} a_{ij}x_j \le b_i + p_i$ ($i \varepsilon N_m$)

$x_j \ge 0$ ($j \varepsilon N_n$).

then, the fuzzy set of optimal values, G, which is a fuzzy subset of R^n, is defined by

$$G(x) = \begin{cases} 1 & when\ z_2 \le cx \\ \frac{cx - z_1}{z_2 - z_1} & when\ z_1 < cx < z_2 \\ 0 & when\ cx \le z_1 \end{cases}$$

Now, the problem becomes the following classical optimization problem:

Max λ

subject to $\lambda(z_2 - z_1) - cx \le -z_1$

$\lambda p_i + \sum_{i=1}^{n} a_{ij}x_j \le b_i + p_i$ ($i \varepsilon N_m$)

$\lambda, x_j \ge 0$ ($j \varepsilon N_n$).

3. PROPOSED PROBLEM

Assume that a company makes two types of products. Product p1 has a Rs.0.40 per unit profit and Product p2 has a Rs. 0.30 per unit profit. Each unit of product p1 requires twice as many labour hours as each product p2. The total available labour hours are at least 500 hours per day, and many possible be extended to 600 hours per day, due to special arrangements for overtime work.

The supply of material is at least sufficient for 400 units of both p1 and p2 products per day, but may possibly be extended to 500 units per day according to previous experience. The problem is show that or find, how many units of product p1 and p2 should be made per day to maximize the total profit.

Solution: First, we calculate the lower and upper bound of the objective function by classical programming method.

Lower bound of the Objective Function :

Let x1, x2 denote the number of units of products p1, p2 made in one day, respectively.

Formulation:

Max Z = 0.4x1 + 0.3 x2

subject to the constraints x1 + x2 $\le$ 400

2x1 + x2 $\le$ 500

and x1, x2 $\ge$ 0

This is in Standard form of LPP.

Now introduce s3 and s4 slack variables to convert the inequality constraints into equality constraints, the formulated new function becomes

Max z1 = 0.4 x1 + 0.3 x2 + 0 s3 + 0 s4

subject to constraints x1+ x2 + s3 = 400

2x1 + x2 + s4 = 500

and x1, x2, s3, s4 $\geq$ 0

By simplex method,

*** Start ***

Basis	X1	X2	s3	s4	RHS
s3	1	1	1	0	400
s4	2	1	0	1	500
Obj.	0.4	0.3	0	0	0

Variable to be made basic -> X1

Ratios: RHS/Column X1 -> { 400 250 }

Variable out of the basic set -> s4

*** Iteration 1 ***

Basis	X1	X2	s3	s4	RHS
s3	0	0.5	1	-0.5	150
X1	1	0.5	0	0.5	250
Obj.	0	0.1	0	-0.2	100

Variable to be made basic -> X2

Ratios: RHS/Column X2 -> { 300 500 }

Variable out of the basic set -> s3

*** Iteration 2 ***

Basis	X1	X2	s3	s4	RHS
X2	0	1	2	-1	300

X1	1	0	-1	1	100
Obj.	0	0	-0.2	-0.1	130

>> Optimal solution FOUND

>> Maximum = 130

*** RESULTS - VARIABLES ***

Variable	Value	Obj. Cost	Reduced Cost
X1	100	0.4	0
X2	300	0.3	0

Hence the optimal solution is x1= 100, x2 = 300 and z1 = 0.4 * 100 + 0.3 * 300 = 130.

Upper bound of the Objective Function :

Max z2 = 0.4x1 + 0.3x2

subject to constraints $x1 + x2 \leq 500$

$2x1 + x2 \leq 600$

and $x1, x2 \geq 0$

Introducing slack variables s3, s4 as the inequality constraints are of type "≤" type

Max Z2 = 0.4x1 + 0.3x2 + 0 s3 + 0 s4

subject to constraints x1 + x2+ s3 = 500

2x1+x2 + s4 = 600

and $x1, x2, s3, s4 \geq 0$

By Simplex method,

*** Start ***

Basis	X1	X2	s3	s4	RHS
s3	1	1	1	0	500
s4	2	1	0	1	600
Obj.	0.4	0.3	0	0	0

Variable to be made basic -> X1

Ratios: RHS/Column X1 -> { 500 300 }

Variable out of the basic set -> s4

*** Iteration 1 ***

Basis	X1	X2	s3	s4	RHS
s3	0	0.5	1	-0.5	200
X1	1	0.5	0	0.5	300
Obj.	0	0.1	0	-0.2	120

Variable to be made basic -> X2

Ratios: RHS/Column X2 -> { 400 600 }

Variable out of the basic set -> s3

*** Iteration 2 ***

Basis	X1	X2	s3	s4	RHS
X2	0	1	2	-1	400
X1	1	0	-1	1	100
Obj.	0	0	-0.2	-0.1	160

>> Optimal solution FOUND

>> Maximum = 160

*** RESULTS - VARIABLES ***

Variable	Value	Obj. Cost	Reduced Cost
X1	100	0.4	0
X2	400	0.3	0

Hence, the optimal solution is x1=100, x2= 400 and z2 = 0.4*100+0.3*400=160.

Optimal Solution : Now, the fuzzy linear programming problem becomes,

Max Z = x3

subject to (160-130)x3 - (0.4x1+ 0.3 x2) $\leq$ -130 or 30 x3 - (0.4x1+0.3 x2)≤ -130

or -30 x3 + (0.4 x1 + 0.3 x2) ≥ 130

100 x3 + x1 + x2 ≤ 500

100 x3+ 2x1 + x2 ≤ 600

and x1, x2,x3 ≥ 0

applying Big-M Method,

Max Z = 0x1 + 0 x2+ x3 +0s4 + 0s5+0s6 - M s7

subject to constraints

-30 x3 + 2/5 x1 + 3/10 x2 - s4 + s7= 130

100 x3 + x1 + x2 + s5=500

100 x3+ 2x1 + x2 + s6= 600

and x1 ,x2, x3, s4, s5, s6,s7 ≥ 0

*** Start ***

Basis	X1	X2	X3	s4	s5	s6	s7	RHS
s7	0.4	0.3	-30	-1	0	0	1	130
s5	1	1	100	0	1	0	0	500
s6	2	1	100	0	0	1	0	600
Obj.	-0.4	-0.3	30	1	0	0	0	130

Variable to be made basic -> X1

Ratios: RHS/Column X1 -> { 325 500 300 }

Variable out of the basic set -> s6

*** Iteration 1 ***

Basis	X1	X2	X3	s4	s5	s6	s7	RHS
s7	0	0.1	-50	-1	0	-0.2	1	10
s5	0	0.5	50	0	1	-0.5	0	200
X1	1	0.5	50	0	0	0.5	0	300
Obj.	0	-0.1	50	1	0	0.2	0	10

Variable to be made basic -> X2

Ratios: RHS/Column X2 -> { 100 400 600 }

Variable out of the basic set -> s7

*** Iteration 2 ***

Basis	X1	X2	X3	s4	s5	s6	RHS
X2	0	1	-500	-10	0	-2	100
s5	0	0	300	5	1	0.5	150
X1	1	0	300	5	0	1.5	250
Obj.	0	0	1	0	0	0	0

Variable to be made basic -> X3

Ratios: RHS/Column X3 -> { - 0.5 5/6 }

Variable out of the basic set -> s5

*** Iteration 3 ***

Basis	X1	X2	X3	s4	s5	s6	RHS
X2	0	1	0	-5/3	5/3	-7/6	350
X3	0	0	1	1/60	1/300	1/600	0.5
X1	1	0	0	0	-1	1	100
Obj.	0	0	0	-1/60	-1/300	-1/600	0.5

>> Optimal solution FOUND

>> Maximum = 0.5

*** RESULTS - VARIABLES ***

Variable	Value	Obj. Cost	Reduced Cost
X1	100	0	0
X2	350	0	0
X3	0.5	1	0

Hence, the optimal solution is x1=100, x2= 350, x3=0.5 and Max Z = 0.4*100+0.3*350=145.

4. CONCLUSION

In this paper, we determine the optimal values by calculating both the lower bound and upper bounds. By converting the Fuzzy LP problem into linear problem, we are evaluated the optimal solution of lower bound and upper bounds. We used Software **Linear Program Solver** to execute the results. Linear Program Solver (LiPS) is intended for solving linear and goal programming problems.

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