Fuzzy Linear Problem Solution Using Lower Bound and Upper Bound Technique

¹SOMESHWAR SIDDI, ²Dr. Y. RAGHUNATHA REDDY

¹Research Scholar, Department of OR & SQC, Rayalaseema University, India

Abstract: Fuzzy Linear problem is an important application of fuzzy set theory in decision making problems and the problems are related to linear programming with fuzzy variables. In this paper, we proposed a method for Fuzzy linear programming problems using lower bound of the objective function and upper bound of the objective function. One numerical example illustrated with the help of the proposed method.

Keywords: Fuzzy linear problems, simplex method, optimal solution, lower bound of objective function, upper bound of objective function.

1. INTRODUCTION

The basic linear programming problem is to find the optimum values of a linear function under given constraints. Linear Programming is one of the most frequently used operations research technique in real time problems. Many real world problems can be transformed into LP model. LP requires much well-defined and precise data which involves high costs. In order to cut the information costs and to avoid unrealistic assumptions, we used fuzzy linear program.

The concept of fuzzy decision making was proposed by **Bellman**, **Zadeh**, in 1970. **Zimmermann** proposed the first method of fuzzy linear programming with several objective functions. **Maleki et al.**, added processes to clear up linear programming issues with fuzzy variables. **Cadenas, Verdegay** have evolved the concept of the usage of fuzzy numbers in linear programming. **Jayalakshmi, Pandian** added a new approach wherein fuzzy hassle is decomposed into 3 crisp linear programming problems. **Lodwick, Bachman** have mentioned huge scale fuzzy feasible optimization issues.

Definition: If $X = \{x\}$ is a collection of objects denoted by x, then a fuzzy set A in X is a set of order pairs

A={x, $\mu_A(x)$: x \in X}, where $\mu_A(x)$ is called the "membership grade" of x in A

Definition: Let A=(a,b), B=(d,e) be two triangular fuzzy numbers then

- (i) A+B = (a+d, b+e)
- (ii) kA = (ka,kb), where k is a real number

Definition: Let A=(a,b) then (i) A is said to be positive if a>0, b>0

(ii) A is said to be integer if $a \ge 0$, $b \ge 0$

2. PROBLEM FORMULATION

The fuzzy LPP is formulated as underneath:

$$\max Z = \sum_{j=1}^{n} C_{j} X_{j}$$

Subject to $\sum_{j=1}^{n} A_{ij} X_{j} \le B_{i}$ (ie N_m)

$$X_i \ge 0$$
 ($j \in N_n$).

where A_{ij} , B_i , C_j are fuzzy numbers, and X_j are variables whose states are fuzzy numbers ($i\epsilon N_m$, $j\epsilon N_n$); the operations of addition and multiplication are operations of fuzzy arithmetic and \leq denote the ordering of fuzzy members.

²Assistant Professor, Department of OR&SQC, Rayalaseema University, India

Vol. 5, Issue 1, pp: (1-8), Month: April - September 2017, Available at: www.researchpublish.com

In this case, fuzzy numbers B_i (i ϵ N_m) typically have the form

$$B_{i}(x) = \begin{cases} 1 & when \ x \le b_{i} \\ \frac{b_{i} + p_{i} - x}{p_{i}} & when \ b_{i} < x < b_{i} + p_{i} \\ 0 & when \ b_{i} + p_{i} \le x \end{cases}$$

where $x \in R$.

Next, we determine the optimal values, by calculating both the lower bound and upper bounds.

Solving LPP to obtain optimal value of z1:

Max Z = cx

subject to
$$\sum_{i=1}^{n} a_{ij} x_i \le b_i$$
 (if N_m)

$$x_i \ge 0$$
 (j εN_n).

Solving LPP to obtain optimal values of z2: replace b_i with $b_i + p_i$,

Max Z=cx

subject to
$$\sum_{i=1}^{n} a_{ii} x_i \le b_i + p_i$$
 (i $\in N_m$)

$$x_i \ge 0$$
 ($j \in N_n$).

then, the fuzzy set of optimal values, G, which is a fuzzy subset of Rⁿ, is defined by

$$G(x) = \begin{cases} 1 & \text{when } z_2 \le cx \\ \frac{cx - z_1}{z_2 - z_1} & \text{when } z_1 < cx < z_2 \\ 0 & \text{when } cx \le z_1 \end{cases}$$

Now, the problem becomes the following classical optimization problem:

Max λ

subject to
$$\lambda(z_2-z_1)-cx \leq -z_1$$

$$\lambda p_i + \sum_{i=1}^n a_{ij}x_j \leq b_i + p_i \ (i\epsilon N_m)$$

$$\lambda, \ x_i \geq 0 \quad (j\epsilon N_n).$$

3. PROPOSED PROBLEM

Assume that a company makes two types of products. Product p1 has a Rs.0.40 per unit profit and Product p2 has a Rs. 0.30 per unit profit. Each unit of product p1 requires twice as many labour hours as each product p2. The total available labour hours are at least 500 hours per day, and many possible be extended to 600 hours per day, due to special arrangements for overtime work.

The supply of material is at least sufficient for 400 units of both p1 and p2 products per day, but may possibly be extended to 500 units per day according to previous experience. The problem is show that or find, how many units of product p1 and p2 should be made per day to maximize the total profit.

Solution: First, we calculate the lower and upper bound of the objective function by classical programming method.

Lower bound of the Objective Function:

Let x1, x2 denote the number of units of products p1, p2 made in one day, respectively.

Formulation:

Max Z =
$$0.4x1 + 0.3 x2$$

subject to the constraints $x1 + x2 \le 400$
 $2x1 + x2 \le 500$
and $x1, x2 \ge 0$

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This is in Standard form of LPP.

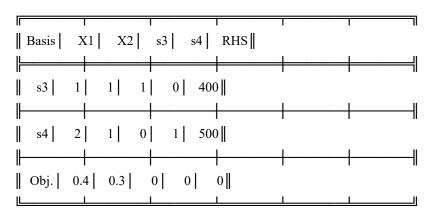
Now introduce s3 and s4 slack variables to convert the inequality constraints into equality constraints, the formulated new function becomes

Max z1 = 0.4 x1 + 0.3 x2 + 0 s3 + 0 s4
subject to constraints
$$x1+ x2 + s3 = 400$$

 $2x1 + x2 + s4 = 500$
and x1, x2, s3, s4 \geq 0

By simplex method,

*** Start ***

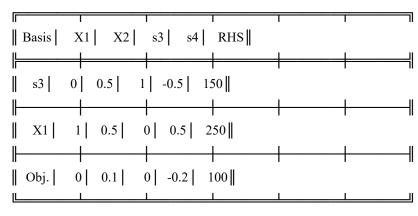


Variable to be made basic -> X1

Ratios: RHS/Column X1 -> { 400 250 }

Variable out of the basic set -> s4

*** Iteration 1 ***

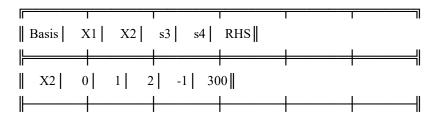


Variable to be made basic -> X2

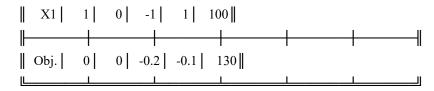
Ratios: RHS/Column X2 -> { 300 500 }

Variable out of the basic set -> s3

*** Iteration 2 ***



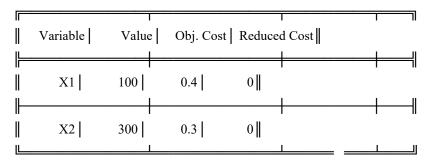
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>> Optimal solution FOUND

>> Maximum = 130

*** RESULTS - VARIABLES ***



Hence the optimal solution is x1=100, x2=300 and z1=0.4*100+0.3*300=130.

Upper bound of the Objective Function:

Max z2 = 0.4x1 + 0.3x2

subject to constraints $x1 + x2 \le 500$

 $2x1 + x2 \le 600$

and x1, $x2 \ge 0$

Introducing slack variables s3, s4 as the inequality constraints are of type "\le " type

$$Max \ Z2 = 0.4x1 + 0.3x2 + 0 \ s3 + 0 \ s4$$

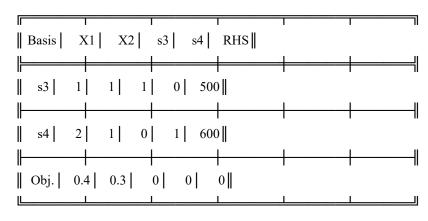
subject to constraints x1 + x2 + s3 = 500

2x1+x2+s4=600

and x1, x2, s3, s4 \geq 0

By Simplex method,

*** Start ***



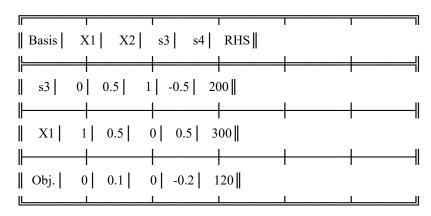
Variable to be made basic -> X1

Ratios: RHS/Column X1 -> { 500 300 }

Variable out of the basic set -> s4

Vol. 5, Issue 1, pp: (1-8), Month: April - September 2017, Available at: www.researchpublish.com

*** Iteration 1 ***

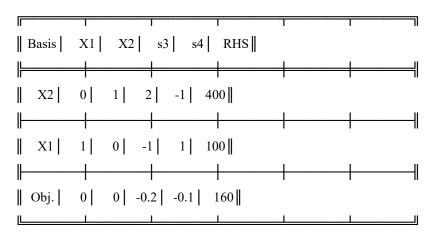


Variable to be made basic -> X2

Ratios: RHS/Column X2 -> { 400 600 }

Variable out of the basic set -> s3

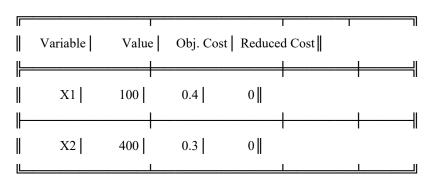
*** Iteration 2 ***



>> Optimal solution FOUND

>> Maximum = 160

*** RESULTS - VARIABLES ***



Hence, the optimal solution is x1=100, x2=400 and z2=0.4*100+0.3*400=160.

Optimal Solution: Now, the fuzzy linear programming problem becomes,

Max Z = x3
subject to
$$(160-130)$$
x3 - $(0.4$ x1+ 0.3 x2) \le -130

Vol. 5, Issue 1, pp: (1-8), Month: April - September 2017, Available at: www.researchpublish.com

or
$$30 \times 3 - (0.4 \times 1 + 0.3 \times 2) \le -130$$

or $-30 \times 3 + (0.4 \times 1 + 0.3 \times 2) \ge 130$
 $100 \times 3 + \times 1 + \times 2 \le 500$
 $100 \times 3 + 2 \times 1 + \times 2 \le 600$
and $\times 1, \times 2, \times 3 \ge 0$

applying Big-M Method,

$$Max\ Z = 0x1 + 0\ x2 + x3\ + 0s4 + 0s5 + 0s6 - M\ s7$$

subject to constraints

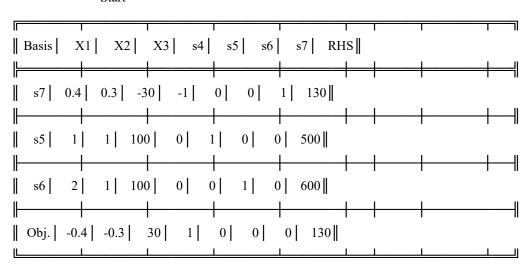
$$-30 x3 + 2/5 x1 + 3/10 x2 - s4 + s7 = 130$$

$$100 x3 + x1 + x2 + s5 = 500$$

$$100 x3 + 2x1 + x2 + s6 = 600$$

and x1,x2, x3, s4, s5, s6,s7
$$\geq$$
 0

*** Start ***

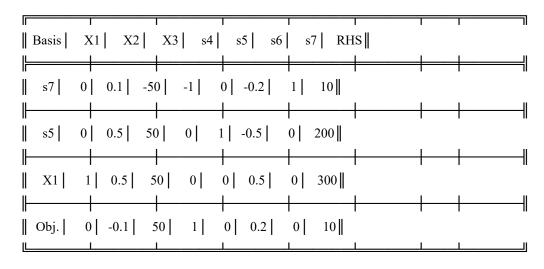


Variable to be made basic -> X1

Ratios: RHS/Column X1 -> { 325 500 300 }

Variable out of the basic set -> s6

*** Iteration 1 ***



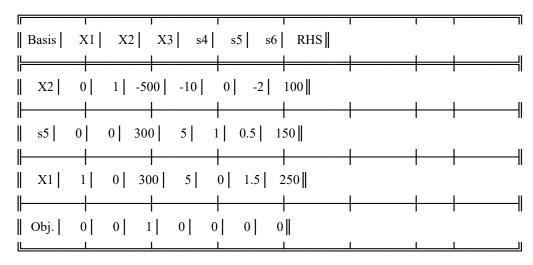
Vol. 5, Issue 1, pp: (1-8), Month: April - September 2017, Available at: www.researchpublish.com

Variable to be made basic -> X2

Ratios: RHS/Column X2 -> { 100 400 600 }

Variable out of the basic set -> s7

*** Iteration 2 ***

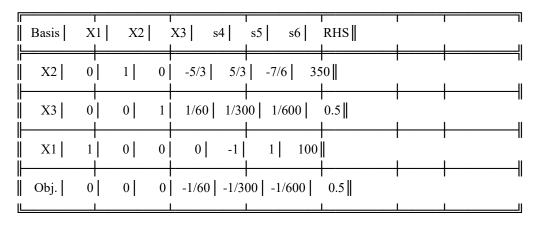


Variable to be made basic -> X3

Ratios: RHS/Column X3 -> { - 0.5 5/6 }

Variable out of the basic set -> s5

*** Iteration 3 ***



>> Optimal solution FOUND

>> Maximum = 0.5

*** RESULTS - VARIABLES ***

Variable	Value	Obj. Cos	st Reduce	l d Cost∥ L	[
X1	100	0	0	I	l
 X2	350	0	0		I I
X3	0.5	1	0		

Hence, the optimal solution is x1=100, x2=350, x3=0.5 and Max Z=0.4*100+0.3*350=145.

Vol. 5, Issue 1, pp: (1-8), Month: April - September 2017, Available at: www.researchpublish.com

4. CONCLUSION

In this paper, we determine the optimal values by calculating both the lower bound and upper bounds. By converting the Fuzzy LP problem into linear problem, we are evaluated the optimal solution of lower bound and upper bounds. We used Software **Linear Program Solver** to execute the results. Linear Program Solver (LiPS) is intended for solving linear and goal programming problems.

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